An alternative NMSSM phenomenology with manifest perturbative unification

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ABSTRACT: Can supersymmetric models with a moderate stop mass be made consistent with the negative Higgs boson searches at LEP, while keeping perturbative unification manifest? The NMSSM achieves this rather easily, but only if extra matter multiplets filling complete SU(5) representations are present at intermediate energies. As a concrete example which makes use of this feature, we give an analytic description of the phenomenology of a constrained NMSSM close to a Peccei-Quinn symmetry point. The related pseudo-Goldstone boson appears in decays of the Higgs bosons and possibly of the lightest neutralino, and itself decays into $b\bar{b}$ and $\tau\bar{\tau}$.

Contents

1. Introduction and motivations

The absence of any clear signal of the Higgs boson(s) at LEP is a disturbing fact for the Minimal Supersymmetric Standard Model (MSSM). As well known a heavy stop could be the explanation. Although possible, however, this weakens the view that requires supersymmetry to be visible at the LHC, especially since the top, and the stop, have the strongest coupling to the Higgs boson system. In the attempt to avoid this quite unpleasant road, several proposals have in fact been made: among them, the consideration of the Next to Minimal Supersymmetric Standard Model (NMSSM) has received a great deal of attention. It is in fact true that the extra contribution to the quartic Higgs coupling arising in the NMSSM can easily accommodate a lightest Higgs boson even much heavier than in the MSSM [1, 2]. A strong constraint, however, to which we stick in this paper, is its compatibility with manifest perturbative unification.

Although the NMSSM is a very minimal extension of the MSSM, it has a drawback: it allows to introduce several more parameters, which often make the various analyses difficult to follow or can even obscure the very search for significant phenomenological patterns. In this paper we try to clarify a possibility offered by the NMSSM to comply with the LEP constraints[1] in a weakly fine-tuned and not too narrow region of its parameter space, while

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[1]For other attempts see e.g. [3−5]. There are, however, two different aspects of the problem that may have not been equally addressed in these works: the level of fine-tuning in the Z-mass and the narrowness of the region of parameter space consistent with current data.

insisting on a relatively light stop. A key point is that such possibility rests on the largest possible values of the usual $\lambda S H_1 H_2$ coupling of the NMSSM consistent with manifest perturbative unification, including the possible existence of extra matter multiplets filling complete SU(5) representations at intermediate energies [8].

With CP conserved in the scalar sector, the NMSSM has three CP even and two CP odd neutral fields. With the standard definition of the Higgs doublets H_1 and H_2, the only scalar with tree level coupling to the vector boson pairs VV, often called h since it is the closest to the Standard Model (SM) Higgs boson, has the composition[2]

$$h = h_1^0 \cos \beta + h_2^0 \sin \beta \tag{1.1}$$

and tree-level diagonal mass-squared

$$(m_h^0)^2 = M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta, \tag{1.2}$$

corrected by the well known radiative contribution from top-stop loops (for moderate mixing, $|A_t/m_{\tilde{t}}| \lesssim 1$)

$$m_h^2 \simeq (m_h^0)^2 + \frac{3m_t^4}{4\pi^2 v^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}. \tag{1.3}$$

We do not show in this radiative correction a small positive contribution due to A_t, because in practice for moderate mixing it is compensated by a further negative $O(\alpha_s \alpha_t)$ correction at the two-loop order. Thus the one-loop result (1.3) remains a reasonable approximation.

Note that equation (1.2) is valid for any scalar potential of the form

$$V = V^{\text{gauge}}(H_1, H_2) + \mu_1^2(S)|H_1|^2 + \mu_2^2(S)|H_1|^2 - (\mu_3^2(S)H_1 H_2 + h.c.) + \lambda^2 |H_1 H_2|^2 + V(S) \tag{1.4}$$

i.e., in particular, for any NMSSM which stays perturbative up to the GUT scale.

The phenomenology of the NMSSM in relation with the Higgs boson searches at LEP certainly depends on the value of m_h, but crucially also on the mixings of h with the two other CP even scalars, since m_h^2, being a diagonal entry of a positive definite squared mass matrix, gives only an upper bound on the mass squared of the lightest physical CP-even scalar.

With this in mind, this paper consists of two logically independent but also complementary parts. In the first one we discuss the maximum possible values of the coupling λ, and therefore of m_h, in presence of extra matter multiplets filling complete SU(5) representations at intermediate energies (section 2). Furthermore, based on the values that we find for m_h, we consider a simple and generic 2×2 mixing model between h and the lightest among the two remaining CP even scalars, which, before mixing with h, do not couple at all with VV (section 3). In the second part we describe a fully detailed and motivated version of the NMSSM with an approximate Peccei-Quinn symmetry that realizes the phenomenological pattern outlined in the first part. This approximate symmetry restricts the number of effective parameters and makes possible an analytic description of most of the relevant features we want to underline.

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[2]As usual, $\tan \beta = v_2/v_1$, $v_i = \langle H_i^0 \rangle$, $v_1^2 + v_2^2 = 174$ GeV.
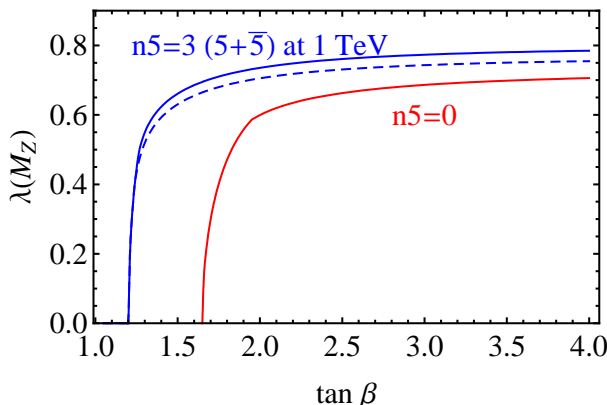
Figure 1: Lower (red) curve: the maximal value of $\lambda(M_Z)$ as a function of $\tan\beta$ in NMSSM without extra matter at intermediate energies, subject to the condition $\lambda_{\mathrm{GUT}}/4\pi < 0.3$, $\kappa_{\mathrm{GUT}} = 0$. Upper (blue) curves: same but with $n_5 = 3$ extra $5 + \bar{5}$ at $1\,\mathrm{TeV}$, and $\lambda_{\mathrm{GUT}}/4\pi < 0.3, 0.15$.

2. On the maximal value of the SH_1H_2 coupling

From eq. (1.2) m_h is especially sensitive to the value of the coupling λ at the weak scale, which is constrained by demanding that λ stays perturbative in its RGE evolution all the way up to the GUT scale. More specifically, since λ grows with the energy from the weak to the GUT scale, we require for its value at the GUT scale, $\lambda_{\mathrm{GUT}} < 0.3 \cdot 4\pi$.

The RGEs of λ and of other relevant couplings can be found in appendix A. A significant indirect effect on the evolution of λ is caused by the presence at intermediate energies of vector-like supermultiplets filling complete SU(5) representations [8]. These multiplets *increase* the gauge couplings at higher energies, which in turn *slows down* the growth of both λ and y_t, delaying the onset of nonperturbative behavior. This effect is illustrated in figure 1 which shows as function of $\tan\beta$ the maximum value of λ at the weak scale without or with extra-matter effects (three $(5 + \bar{5})$ of SU(5) at the weak scale), for the current value of $m_t = 171\,\mathrm{GeV}$.[3] Consequently, from eqs. (1.2), (1.3), figure 2 gives the maximum value of m_h for a moderate stop mass, $m_{\tilde{t}} = 300\,\mathrm{GeV}$. The upper blue curves, for $\lambda_{\mathrm{GUT}}/4\pi = 0.3, 0.15$ are again with three $(5 + \bar{5})$ of SU(5) at the Fermi scale, whereas the lower red curve, for $\lambda_{\mathrm{GUT}}/4\pi = 0.3$, includes in the RGE evolution the standard matter effects only.

Several features in these figures are worth being observed. All the curves in figure 2 go for large $\tan\beta$ to a common asymptotic value which is the upper bound on m_h in the MSSM (for the same $m_{\tilde{t}} = 300\,\mathrm{GeV}$ and moderate mixing). Relative to this value, the increment in m_h due to the extra three $(5 + \bar{5})$ is clearly significant, especially since without extra matter the maximum value of m_h barely touches the LEP bound on the SM Higgs boson mass of about $115\,\mathrm{GeV}$. This is even more so since the upper limit on m_h is essentially saturated for wide variations of λ_{GUT}, in its upper range, as shown by the close upper curves in figure 2.

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[3]The example $n_5 = 4$ at $1\,\mathrm{TeV}$ emphasized in [8] gives non-perturbative values of α_{GUT} once the 2-loop terms are included in gauge beta functions.
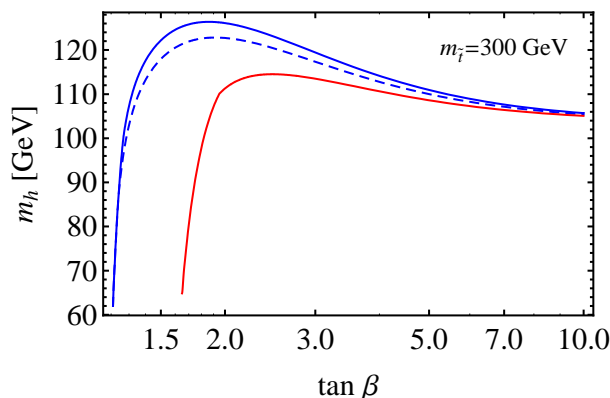
Figure 2: The maximal value of m_h, see eq. (1.3), in NMSSM without extra matter (red) and with $n_5 = 3$ extra $5 + \bar{5}$ at 1 TeV (blue). The values of λ_{GUT} are the same as in figure 1. The stop mass is fixed at $m_{\tilde{t}} = 300\,\mathrm{GeV}$ with moderate mixing, $|A_t/m_{\tilde{t}}| \lesssim 1$.

Both figure 1 and 2 are for vanishing $\frac{\kappa}{3}S^3$ coupling in the superpotential, but they are all insensitive to any choice of $\kappa_{\mathrm{GUT}}/4\pi \leq 0.05$, since κ is rapidly driven to zero at lower energies by the RGE evolution. A larger κ_{GUT} would however reduce the maximum λ at the weak scale.

Finally notice that the curves without any extra matter start at $\tan\beta \simeq 1.6$ because at lower $\tan\beta$, unlike in the case with extra matter, the top Yukawa coupling hits by itself the perturbative bound of $0.3 \cdot 4\pi$ before getting to the unification scale.

Recently, Dine, Seiberg and Thomas [10] have claimed that in a singlet extension of MSSM based on the superpotential $\mu H_1 H_2 + \lambda S H_1 H_2 + \frac{1}{2}MS^2$ one can raise the Higgs mass by a significant amount while maintaining manifest perturbative unification *without extra matter at intermediate scales*. E.g., one of their examples ([10], section 4.1) had $m_h = 120\,\mathrm{GeV}$ for $\tan\beta = 4$, $\lambda = 0.7$, $m_{\tilde{t}} = 300\,\mathrm{GeV}$[4] and no mixing, which is in clear contradiction with figure 2. As is stressed in the Introduction, our bound on m_h applies to any Higgs potential of the form (1.4), and in particular to the superpotential of [10]. We believe that the expansion analysis of [10], based on integrating out S and analyzing the spectrum of light states in terms of coefficients of dimension 6 operators, must be breaking down, and this explains the discrepancy.

We conclude this section by analyzing the effect of extra SU(5) multiplets on the gauge coupling unification. In table 1 we show the prediction of $\alpha_S(M_Z)$ for $n_5 = 3$ from the running of the gauge couplings at one and two loops, compared with the standard case ($n_5 = 0$), without any threshold effect. In the same table we give, for the two cases, the corresponding value of the unified coupling α_G. As is well known, the one loop prediction is very close to the experimental value $\alpha_S(M_Z) = 0.1176(20)$, and of course this conclusion is left unchanged by the addition of extra matter in full SU(5) multiplets. At two loops, the prediction for $n_5 = 3$ is brought closer to the experiment compared to the standard

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[4]In fact they used the soft stop mass of $300\,\mathrm{GeV}$, which corresponds to physical stop mass of $345\,\mathrm{GeV}$, an irrelevant difference.

$n_5 = 0$		$n_5 = 3$		
$\alpha_S(M_Z)$	α_G	$\alpha_S(M_Z)$	α_G	
0.117	0.041	**0.117**	0.103	1-loop
0.130	0.043	**0.123**	0.154	2-loop numerical
0.129	0.043	**0.122**	0.143	2-loop analytical

Table 1: Prediction for $\alpha_S(M_Z)$ in the standard case ($n_5 = 0$) and for $n_5 = 3$ ($5+\bar{5}$) at 1 TeV. We use one and two loop gauge beta functions given in appendix A *without the two loop contributions of* λ *and* y, which can later be included perturbatively. The input $\overline{\text{MS}}$ values are $\hat{\alpha}(M_Z)^{-1} = 127.918$, $\sin^2 \hat{\theta}_W(M_Z) = 0.23122$. We do not include any threshold corrections. The last line of the table is obtained by treating the 2-loop terms as perturbative corrections to the 1-loop results, following the standard method as described e.g. in [11].

$n_5 = 0$ result. However, the unavoidable presence of threshold corrections does not allow a significant distinction between the two cases. In fact, a i-th ($5+\bar{5}$) split into a SU(3)-triplet of mass M_{di} and a SU(2) doublet of mass M_{Li}, $\alpha_S(M_Z)$ gives a further one loop threshold correction

$$\frac{\delta\alpha_S(M_Z)}{\alpha_S(M_Z)} \simeq \frac{9\alpha_S(M_Z)}{14\pi} \log \frac{M_{di}}{M_{Li}} \approx 2\% \ \log \frac{M_{di}}{M_{Li}} \ . \tag{2.1}$$

There is furthermore a two loop contribution from λ itself, dominated by the UV,

$$\frac{\delta\alpha_S(M_Z)}{\alpha_S(M_Z)} \simeq -\frac{9\alpha_S(M_Z)}{56\pi} \log \left(\frac{\lambda_{GUT}^2}{2\pi^2} \log \frac{M_{\text{GUT}}}{M_Z} \right), \tag{2.2}$$

i.e., numerically, $\delta\alpha_S(M_Z)/\alpha_S(M_Z) \simeq -(1 \div 2)\%$ for $\lambda_{\text{GUT}}/4\pi = 0.3 \div 0.2$.

3. A simple 2×2 mixing model

As already mentioned, what really matters for the NMSSM phenomenology, more than m_h itself, are the masses and compositions of the physical scalars. Before mixing with h, the MSSM has two other CP-even fields, s_1, s_2 (in their mass-squared diagonal basis). Both their masses and compositions depend on all the various parameters of the NMSSM. Nevertheless, none of them is coupled to VV.

Mixing of h with s_1 and s_2 (if one, or perhaps both, of these states are lighter than h) can help increase the mass of h. After the mixing, s_1 and s_2 acquire coupling to VV and become subject to LEP searches. As we are going to see, these mixing cannot be large for consistency with LEP. As such, one can analyze individually their additive effects without making any significant error. We can then consider a simplified 2×2 mixing model[5] between h and the lightest, s_1, among the two states not coupled to ZZ. Thus we consider a mass matrix

$$\mathcal{M}_{2\times 2}^2 = \begin{pmatrix} m_h^2 & \Delta m^2 \\ \Delta m^2 & m_{s_1}^2 \end{pmatrix} \tag{3.1}$$

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[5]The effects of a 2×2 mixing model of this type has already been considered in the NMSSM [9, 5] and in the MSSM as well [12−14],[15].
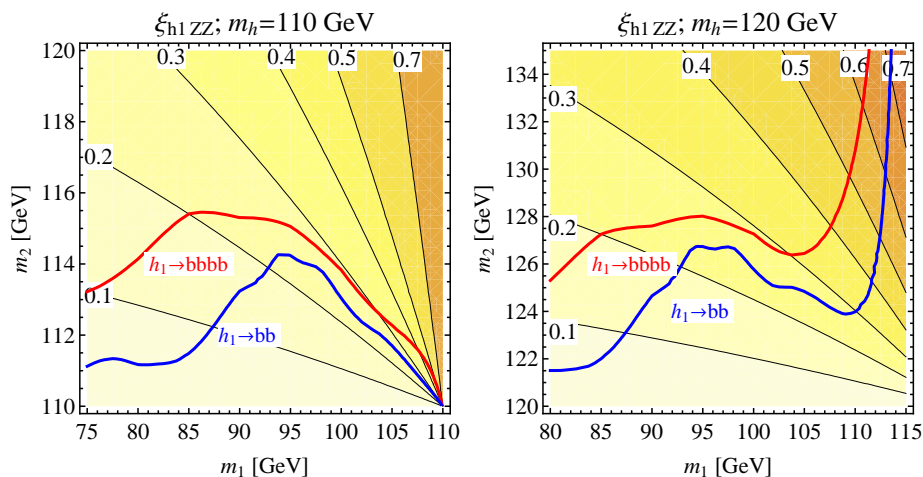
Figure 3: For two reference values $m_h = 110\,\mathrm{GeV}$ (left) and $m_h = 120\,\mathrm{GeV}$ (right) we plot the normalized squared coupling, eq. (3.2), of the lightest scalar h_1 in the 2×2 mixing model eq. (3.1), plotted as a function of its mass m_1 and the mass of the heavier state m_2. The region below the lower blue (upper red) curve is consistent with the 95% C.L. bounds [16] from nonobservation of h_1 at LEP2, assuming it decays into $b\bar{b}$ ($b\bar{b}\,b\bar{b}$). The heavier scalar h_2, to be consistent with the LEP2 searches, should have the mass above $\sim 114\,\mathrm{GeV}$.

with a fixed m_h and arbitrary $m_{s_1} < m_h$ and Δm^2.

In view of the previous section and having in mind the LEP bound of about $115\,\mathrm{GeV}$, only valid for the SM Higgs boson, we take for m_h two reference values, 110 and $120\,\mathrm{GeV}$, close to the upper bounds on m_h without or with extra matter respectively. In absence of mixing only the latter case would be compatible with LEP data. With mixing, however, which is generally present, the situation may change.

In figures 3, we describe the effect of mixing h with s_1 in the two cases. In the plane of the two mass eigenvalues (m_1, m_2) — from which we can uniquely reconstruct $m_{s_1}^2$ and Δm^2 — we give the isolines of the squared coupling of the lightest state to ZZ, normalized to the SM Higgs boson coupling:

$$\xi_{h_1 ZZ} = \left(\frac{g_{h_1 ZZ}}{g_{hZZ}} \right)^2 . \tag{3.2}$$

From the data of ref. [16] this allows to determine in the same plane the 95% C.L. bound from the non-observation of the lightest state, assumed to decay in $b\bar{b}$ with SM branching ratio. For later purposes we also consider the decay in $b\bar{b}\,b\bar{b}$ with a branching ratio close to 1. Given the actual numbers, a quick way to understand from these figures the compatibility with LEP data is to see if there are values of the heaviest mass m_2 above $115\,\mathrm{GeV}$ and simultaneously allowed by the bound on the lightest state.

The conclusions are quite clear. With an unmixed value of $m_h = 110\,\mathrm{GeV}$, and *a fortiori* for lower values, it is hardly possible to obtain consistency with the LEP data.[6]

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[6]However, notice the point at $(m_1, m_2) \approx (95, 115)\,\mathrm{GeV}$ which has been emphasized in the literature [13] in connection with a slight excess of events at LEP.

This means that, with a moderate stop mass and a small A_t-term, the NMSSM *without extra matter* and with standard Higgs boson decays can perhaps be accommodated with LEP data, if at all, only in a small corner of its parameter space. This may explain the interest of considering the decay of the lightest state into $\tau\bar{\tau}\,\tau\bar{\tau}$, which is experimentally less constrained [3].

On the other hand, the $m_h = 120\,\text{GeV}$ case is obviously compatible with LEP data for small enough mixing. More important is that some mixing effects will inevitably be present, which can push the heavier state even further up with a somewhat reduced coupling to the ZZ, while keeping consistency with the LEP data for the lower state. This can be a characteristic feature of the NMSSM with extra matter contributing to the RGE running of the coupling constants, and is the phenomenological pattern to which we want to draw attention.

4. An explicit example based on an approximate Peccei-Quinn symmetry: PQ SUSY

4.1 The Lagrangian and the allowed parameter space

An independent motivation for the NMSSM is that it may provide a simple solution of the so called μ-problem: the supersymmetric superpotential mass term $\mu H_1 H_2$ gets replaced by $\lambda\langle S\rangle H_1 H_2$ and all the mass terms in the Lagrangian originate from supersymmetry breaking. This possible solution of the μ-problem invites a symmetry explanation of the absence of mass terms in the superpotential. Such symmetries can be a continuous R-invariance and/or a Peccei-Quinn (PQ) symmetry. In this paper we choose a PQ symmetry since: i) it removes the $\frac{\kappa}{3}S^3$ coupling, thereby helping to maximize λ at the weak scale; ii) it can reduce the number of parameters in the supersymmetry breaking Lagrangian as well, since PQ may be approximately realized in this sector without conflicting with experiments. This version of NMSSM, which we call "PQ SUSY," has a minimal number of parameters, and contains a light pseudo-Goldstone boson. For earlier considerations of NMSSM in the PQ limit, see [6, 7].

Up to the small breaking of the PQ symmetry, the Lagrangian is uniquely fixed by the superpotential term

$$f = \lambda S H_1 H_2, \tag{4.1}$$

by the soft non-supersymmetric piece of the scalar potential

$$V_{\text{soft}} = m_S^2 |S|^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + (A_\lambda \lambda S H_1 H_2 + \text{H.c.}), \tag{4.2}$$

and by the gaugino mass terms, which we shall take large relative to $\lambda\langle S\rangle$ (see below). Small breaking terms of the PQ symmetry, like $\delta V = m^2 S^2 + B\mu H_1 H_2 + \text{H.c.}$, will have to be present. However we assume them to be small enough only to give mass to the otherwise massless pseudo-Goldstone boson, without significantly affecting any of the remaining properties of the model. We have checked that this is a consistent approximation.
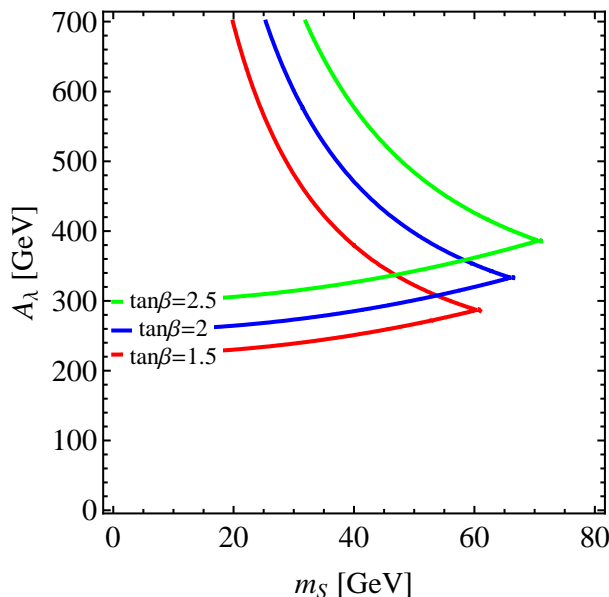
Figure 4: The allowed region of the (m_S, A_λ) plane (see the text) for $\tan\beta = 1.5, 2, 2.5$ and λ fixed at $0.65, 0.7, 0.75$, respectively.

When it exists, the CP-conserving, $\mathrm{SU}(2) \times \mathrm{U}(1) \to \mathrm{U}(1)$ breaking vacuum is related to the Lagrangian parameters by $(x = m_S^2/\lambda^2 v^2)$

$$\lambda^2 v^2 = M_Z^2 + \frac{A_\lambda^2}{1+x} + \frac{m_1^2 - m_2^2}{\cos 2\beta}, \tag{4.3}$$

$$\sin^2 2\beta = 2\left[(1+x) - (1+x)^2 \frac{m_1^2 + m_2^2 + \lambda^2 v^2}{A_\lambda^2}\right], \tag{4.4}$$

$$\langle S \rangle \equiv v_s = \frac{A_\lambda}{2\lambda(1+x)} \sin 2\beta. \tag{4.5}$$

Note that the scalar sector defined by eqs. (4.1), (4.2) depends upon five parameters (apart from the pseudo-Goldstone mass m_G): λ and $m_1^2, m_2^2, m_S^2, A_\lambda$. We trade $m_{1,2}^2$ for v and $\tan\beta$. A useful way to represent the various results, which we shall follow, is to show them in the plane (m_S, A_λ) for fixed values of $\lambda, \tan\beta$. In particular the vacuum in eqs. (4.3), (4.4), (4.5) is indeed the true minimum of the overall potential only in a portion of this parameter space. Figure 4 shows the allowed parameter space for $\tan\beta = 1.5, 2, 2.5$ and λ close to the maximal allowed values from figure 1. We see that A_λ has a maximal and minimal allowed value for each m_S in an interval $0 < m_S < m_S^{\max} \simeq 70\,\mathrm{GeV}$. Here $m_S^2 > 0$ is required by the global stability. The upper limit on A_λ comes from imposing that $V < 0$ at the minimum (4.3), (4.4), (4.5), so that it is preferred to the trivial stationary point at $v_1 = v_2 = v_s = 0$. The lower limit on A_λ comes from the experimental bound on the chargino mass, $m(\chi^\pm) > 103\,\mathrm{GeV}$ [17], via eq. (4.14) below. The constraint of local stability does not further restrict the parameter space.

4.2 Higgs boson and higgsino spectra

The spectrum of the Higgs boson sector is straightforwardly obtained by expanding around the above minimum. For the single charged boson one finds

$$m_{H^\pm}^2 = M_W^2 + \frac{A_\lambda^2}{1+x} - \lambda^2 v^2. \tag{4.6}$$

Out of the two neutral CP-odd states, one is massless in this approximation (the PQ Goldstone G) and the other has mass

$$m_A^2 = \frac{A_\lambda^2}{1+x} + \lambda^2 v^2 (1+x). \tag{4.7}$$

Their compositions in terms of the neutral fields

$$H_1^0 = \frac{1}{\sqrt{2}}(h_1 + i\pi_1), \ H_2^0 = \frac{1}{\sqrt{2}}(h_2 + i\pi_2), \ S = \frac{1}{\sqrt{2}}(s + i\pi_s), \tag{4.8}$$

are

$$G = -\sin\alpha \pi_s + \cos\alpha(\cos\beta \pi_2 - \sin\beta \pi_1), \tag{4.9}$$

$$A = \cos\alpha \pi_s + \sin\alpha(\cos\beta \pi_2 - \sin\beta \pi_1), \tag{4.10}$$

where

$$\tan\alpha = \frac{A_\lambda}{\lambda v(1+x)}. \tag{4.11}$$

The 3×3 squared mass matrix of the CP-even neutral scalars is best written in the basis

$$(H = \cos\beta h_2 - \sin\beta h_1, \ h = \cos\beta h_1 + \sin\beta h_2, \ s) \tag{4.12}$$

where it has the form

$$\mathcal{M}^2 = \begin{pmatrix} \frac{A_\lambda^2}{1+x} + (M_Z^2 - \lambda^2 v^2)\sin^2 2\beta & -\frac{1}{2}(M_Z^2 - \lambda^2 v^2)\sin 4\beta & -A_\lambda \lambda v \cos 2\beta \\ * & M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & -A_\lambda \lambda v \sin 2\beta \frac{x}{1+x} \\ * & * & \lambda^2 v^2(1+x) \end{pmatrix}. \tag{4.13}$$

Note, as anticipated, that $\mathcal{M}_{22}^2 = (m_h^0)^2$. Note also that one of the mixing terms between h and the two other scalars, $\mathcal{M}_{12}^2$ is always small, whereas the other, $\mathcal{M}_{23}^2$, is essentially controlled by x (or m_S^2). The mixing pattern discussed in the previous section is precisely realized in this case, as shown in figure 5, where one has the two lightest scalar masses as functions of m_S for $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400 \, \text{GeV})$. The lightest scalar mass is below the LEP limit; its dominant decay mode (see the next section) is into 2 PQ pseudo-Goldstones: $S_1 \to GG \to 4b$. From figure 6 we see that for $(\lambda, \tan\beta) = (0.7, 2)$ the LEP constraint on the S_1 coupling to ZZ is satisfied in most of the parameter space allowed by the potential stability and the chargino mass bound.[7] The heaviest CP-even scalar has mass $m_{S_3} \approx A_\lambda/(1+x)^{1/2}$ which for $A_\lambda = 400 \, \text{GeV}$ is in the $380 \div 400 \, \text{GeV}$ range, while A and $H^\pm$ are $\sim 15 \, \text{GeV}$ heavier and lighter than S_3, respectively.
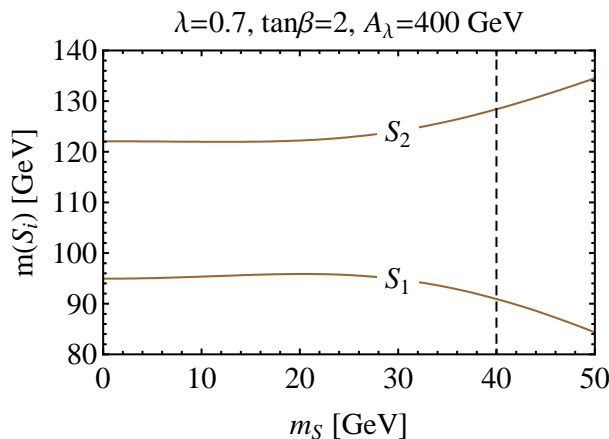
Figure 5: The masses of the two lightest CP-even scalars in the model of section 4 for $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400 \text{ GeV})$ and a range of m_S values consistent with global stability of the scalar potential. We include the stop quantum correction with $m_{\tilde{t}} = 300 \text{ GeV}$. The region to the right of the dashed line ($m_S > 40 \text{ GeV}$) is excluded by LEP constraints on the $S_1 ZZ$ coupling (see figure 6).
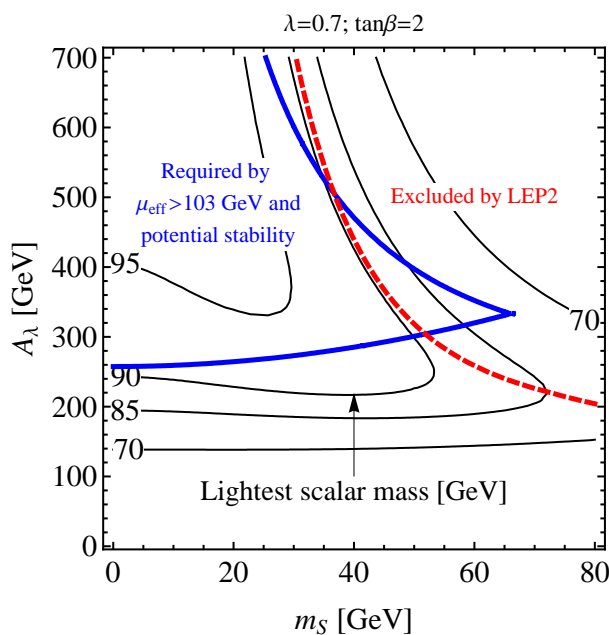

Figure 6: For $(\lambda, \tan\beta) = (0.7, 2)$ we give the allowed regions in (m_S, A_λ) plane following from the stability of the potential and the chargino mass bound (inside the blue curve, same as in figure 4), and from the non-observation of $S_1 \to GG \to 4b$ decays at LEP2 (to the left of the dashed red curve). In the same plot we show contours of the lightest CP-even scalar mass. As always, we include the stop quantum correction with $m_{\tilde{t}} = 300 \text{ GeV}$.

For heavy gaugino masses, the masses of the higgsinos are controlled by the effective

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[7]The processes $e^+ e^- \to Z^* \to S_{1,2} G$ followed by $S_{1,2} \to GG$ could not possibly be seen at LEP2: the normalized squared couplings $\xi_{Z S_{1,2} G}$ are tiny, $\lesssim 10^{-2}$, one order of magnitude below the LEP2 limits [16].
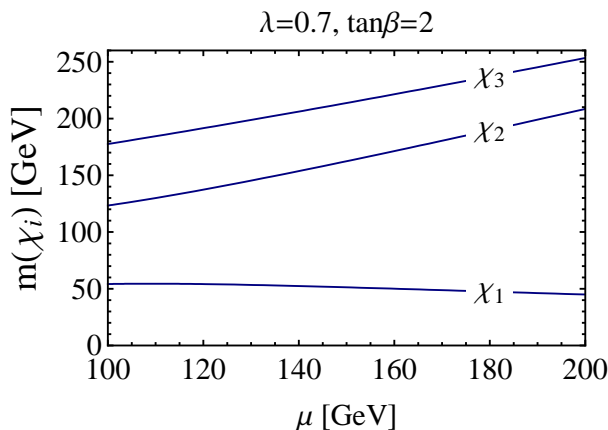
Figure 7: The neutralino masses in the model of section 4 for $(\lambda, \tan\beta) = (0.7, 2)$ and the chargino mass μ in its typical range. We have $m(\chi_1) + m(\chi_2) = m(\chi_3)$ since the neutralino mass matrix (4.15) is traceless.

μ-parameter

$$\mu = \lambda v_s = \frac{A_\lambda}{2(1+x)} \sin 2\beta. \tag{4.14}$$

The single charged higgsino has mass $m(\chi^\pm) = \mu$, whereas the 3×3 neutralino mass matrix in the basis $(\tilde{h}_1, \tilde{h}_2, \tilde{s})$ has the form

$$\mathcal{M}_\chi = \begin{pmatrix} 0 & \mu & \lambda v \sin\beta \\ * & 0 & \lambda v \cos\beta \\ * & * & 0 \end{pmatrix}. \tag{4.15}$$

Figure 7 shows the values of the neutralino masses versus $m(\chi^\pm)$ in its typical range, $100 \div 200 \text{GeV}$, for $(\lambda, \tan\beta) = (0.7, 2)$. For these masses the LEP2 searches have not been possibly effective. Indeed, the process $e^+ e^- \to Z^* \to \chi_1 \chi_2$ is within the LEP2 kinematic limit in a part of the parameter space (see figure 7). However, the production cross section turns out to be well below the ~ 0.1 pb limit set in [17] due to phase space and coupling suppressions.

Another possible process is $e^+ e^- \to \chi_1 \chi_1 \gamma_{\text{ISR}}$ with a photon (from Initial State Radiation) and missing transverse energy in the final state, which is constrained by LEP2 searches of extra neutrino species [18]. However, we concluded that the existing data cannot rule out a χ_1 with a somewhat reduced $Z\chi_1\chi_1$ coupling and mass above $m_Z/2$, as it is in our case (see also [19]).

If the gravitino is the lightest SUSY particle, the lightest neutralino will predominantly decay into the gravitino and the pseudo Goldstone boson G. However, if the SUSY breaking scale $\sqrt{F}$ exceeds about 1000TeV, these decays happen outside the detector and do not modify collider phenomenology of the model (see section 5).

4.3 Higgs boson couplings and branching ratios

The phenomenology of the model is made peculiar by the presence of the light pseudoscalar

	Production coupling	Branching ratios
S_1	$\xi_{S_1 tt}, \xi_{S_1 VV} \lesssim 20\%$ (figure 8)	$BR(GG) \geq 98\%)$
S_2	$\xi_{S_2 tt}, \xi_{S_2 VV} \simeq 100\%$	See figure 9: $BR(\chi_1 \chi_1) = 50 \div 90\%$ $BR(GG) \simeq 1 - BR(\chi_1 \chi_1)$
S_3	$\xi_{S_3 tt} \simeq 20\%$, $\xi_{S_3 VV}$ negligible	See figure 9: $BR(\chi_i \chi_j) \simeq 35\%$ (of which 50% into $\chi_1 \chi_1$) $BR(ZG) \simeq 30\%$ $BR(S_i S_j) \simeq 20\%$

Table 2: The neutral CP-even Higgs boson dominant decay modes and the couplings relevant for their production at the LHC via gluon fusion and vector boson fusion processes, for $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400\,\text{GeV})$ and $0 < m_S \lesssim 40\,\text{GeV}$.

G, with an unknown mass, m_G, coming from the breaking of the PQ symmetry and assumed to be relatively small.

If m_G is below the $b\bar{b}$ threshold, the pseudoscalar can be seen in radiative Υ decays [20]. The relevant branching ratio is given by [21]

$$BR(\Upsilon \to \gamma G) = F(\cos\alpha)^2 (\tan\beta)^2 \frac{G_F m_b^2}{\sqrt{2}\pi\alpha_{\text{EM}}} BR(\Upsilon \to \mu^+\mu^-)$$
$$\simeq 2 \times 10^{-4} F(\cos\alpha)^2(\tan\beta)^2, \quad F = \left(1 - \frac{m_G^2}{m_\Upsilon^2}\right) F_0,$$

where α is the angle in (4.11), and the suppression factor $F_0 \lesssim 0.5$ is due to QCD, bound state and relativistic corrections (see [22], section 3.1). For $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400\,\text{GeV})$ we get $BR(\Upsilon \to \gamma G) \simeq 0.5 \times 10^{-5}$. The experimental limits on this branching ratio depend crucially on the decay properties of G. An interesting possibility occurs if $2m_\tau < m_G < 2m_b$, so that G decays into $\tau^+\tau^-$. In this case the current limit from CLEO is [23]

$$BR(\Upsilon \to \gamma G(\to \tau\tau)) \lesssim 10^{-4},$$

and a dedicated run by BABAR may improve it soon by $1-2$ orders of magnitude.

Below we will assume that m_G is above the $b\bar{b}$ threshold, corresponding to a relatively less restricted region of parameter space. The pseudoscalar then decays into $b\bar{b}$ and $\tau\bar{\tau}$ with branching ratios close to the branching ratios of the SM Higgs boson.

All the couplings and decay rates for the other Higgs bosons are easily determined from the parameters of the model as given in the previous section. Table 2 and figures 8, 9 illustrate the main features of the most relevant quantities for $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400\text{ GeV})$ and $0 < m_S \lesssim 40\,\text{GeV}$ (see figure 6).

Using these numbers one can make a preliminary conclusion that observing these states at the LHC will not be easy, since the production cross sections are suppressed, and the dominant decay products do not allow for easy background discrimination. Obviously, a more detailed study is required to assess the LHC discovery potential.
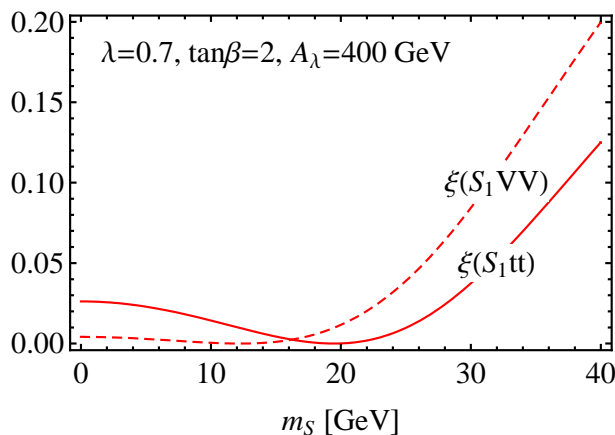
Figure 8: The $S_1 t\bar{t}$ and $S_1 V_\mu V^\mu$ couplings squared of the lightest CP-even scalar S_1, normalized to the couplings of the SM Higgs boson analogously to eq. (3.2). In this plot $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400\text{GeV})$.
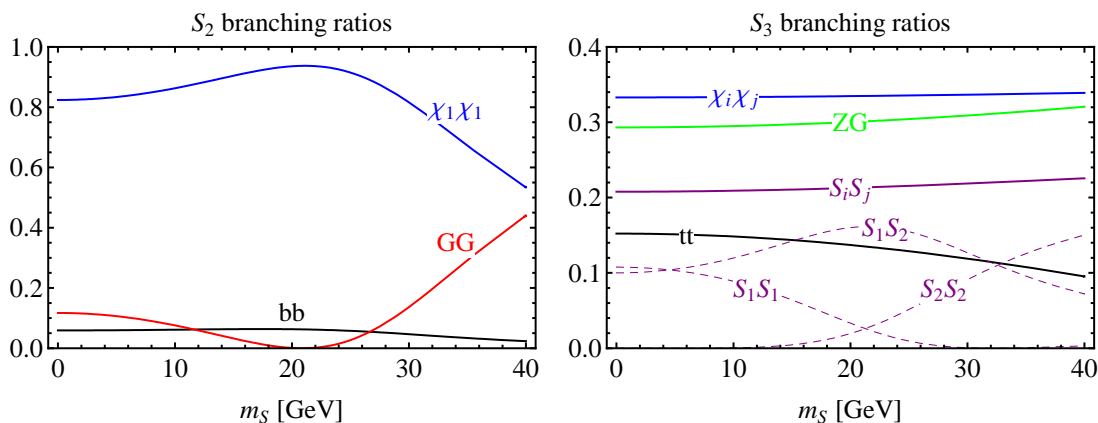

Figure 9: The dominant branching ratios of S_2 (left) and S_3 (right) for $(\lambda, \tan\beta, A_\lambda) = (0.7, 2, 400\text{GeV})$.

4.4 Fine tuning

It is interesting to know an estimate of the finetuning required to satisfy the various restrictions on the parameters of the model under consideration. For an early discussion of finetuning in the NMSSM see [24]. The first thing to check is if there is too strong dependence of the Z-mass, or of the vacuum expectation value v in eq. (4.3), on the various parameters. The strongest such dependence is on the parameter A_λ^2. Naively this would seem to require finetuning of the order $A_\lambda / \lambda^2 v^2 \sim 10$ for $A_\lambda = 400\,\text{GeV}$. However, this estimate does not take into account the fact that the two cancelling terms are not totally independent: the variation of A_λ influences the other term via the angle β as determined by the second equation (4.4). An estimate which takes this effect into account is given by

the logarithmic derivative[8]

$$\Delta = \left| \frac{\partial \log v^2}{\partial \log A^2} \right| , \qquad (4.16)$$

which can be evaluated numerically, see figure 10. We see that $\tan\beta$ below ~ 1.7 starts to be disfavored by this finetuning, although $\tan\beta = 2$, $A_\lambda = 400\,\mathrm{GeV}$ is OK with less than 10% finetune.

Whereas these considerations apply to the dependence of v on the low energy parameters, it is also necessary to check the consistency of the values of these same parameters with the expected contributions due to RGE evolution.[9] Here the difference $m_2^2 - m_1^2$ gets a contribution due to the stop mass:

$$\delta(m_2^2 - m_1^2) = \frac{3y_t^2}{4\pi^2}(m_{\tilde{t}}^2 - m_t^2) \log \frac{\Lambda_{\mathrm{mess}}}{v} \ (\text{no mixing}) \qquad (4.17)$$

where $\log \Lambda_{\mathrm{mess}}/v = 6 \div 40$ for $\Lambda_{\mathrm{mess}} = 100\,\mathrm{TeV} \div 10^{19}\,\mathrm{GeV}$. This equation can be used to set an upper bound on the stop mass with no finetuning, i.e. for $\delta(m_2^2 - m_1^2)/(m_2^2 - m_1^2) = 1$, see figure 11, where we take low mediation scale, $\Lambda_{\mathrm{mess}} = 100\,\mathrm{TeV}$.[10] We see that $m_{\tilde{t}} = 300\,\mathrm{GeV}$, as assumed above, is safely within the allowed range.

Finally we notice that, although v in eq. (4.3) is only weakly dependent on m_S^2, the current experimental constraints mostly on the chargino mass (see eq. (4.14) and figure 4) require m_S to be below $40 \div 50\,\mathrm{GeV}$. At the same time there is a one-loop contribution to the running of m_S^2 due to A_λ^2:

$$\delta m_S^2 = \frac{1}{4\pi^2} \lambda^2 A_\lambda^2 \log \frac{\Lambda_{\mathrm{mess}}}{v} \qquad (4.18)$$

For $\lambda = 0.7$, $A_\lambda = 400\,\mathrm{GeV}$, and $\Lambda = 100\,\mathrm{TeV}$ this gives $\delta m_S^2 = (110\,\mathrm{GeV})^2$. Since the allowed range of m_S^2 is a factor $5 \div 10$ smaller, to comply with this limit it is clear that the model under consideration would again prefer low mediation scale.

5. Conclusions and outlook

Even if we assume, with good reasons indeed, that supersymmetry is relevant in nature, there is no water-tight argument that requires the presence of supersymmetric signals at the LHC. Our best hope is a natural solution of the hierarchy problem of the Fermi scale, which makes such a presence likely. For this to be the case, however, requires a low level of fine tuning in the Z-mass, i.e. a maximally natural solution of the hierarchy problem. The LEP limit on the Higgs boson mass is particularly important, since it excludes the most natural regions of parameter space of the simplest supersymmetric models. This amply motivates the focus on supersymmetric extensions of the SM that minimize this fine-tuning and remain, at the same time, reasonably simple.

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[8]As a consequence of the above effect the derivative $\partial v^2/\partial A^2$ is actually *negative*.

[9]Alternatively we could look directly at the dependence of v on the high energy paremeters, which are considered more fundamental.

[10]To make this plot, parameters m_1^2 and m_2^2 have to be expressed in terms of $v, \lambda, \tan\beta, A_\lambda$ and m_S from equations (4.3), (4.4).
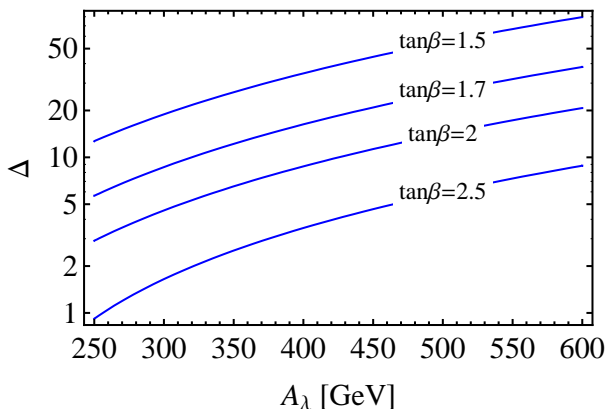
Figure 10: The logarithmic derivative eq. (4.16) plotted as a function of A_λ for $(\tan\beta, \lambda) = (1.5, 0.65), (1.7, 0.7), (2, 0.7), (2.5, 0.75)$. Smaller $\tan\beta$ give bigger Δ (and hence require bigger fine-tuning). In this plot we assumed $m_S = 0$, but the change for small allowed values of m_S is negligible.
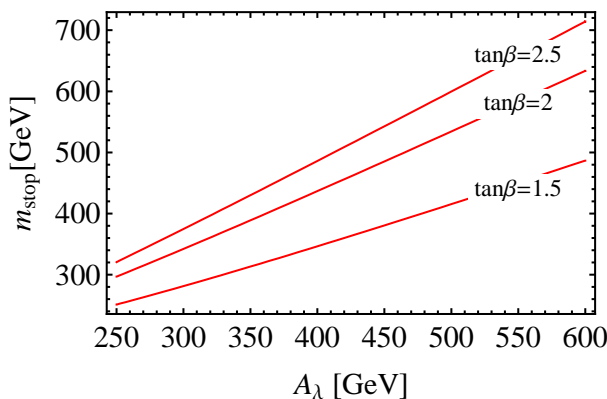

Figure 11: The maximal value of physical stop mass (with no mixing) which is consistent with the naturalness bound $\delta(m_2^2 - m_1^2)/(m_2^2 - m_1^2) < 1$, see eq. (4.17). From below up: $(\tan\beta, \lambda) = (1.5, 0.65), (2, 0.7), (2.5, 0.75)$. Mediation scale $\Lambda_{\text{mess}} = 100\,\text{TeV}$. In this plot we assumed $m_S = 0$, but the change for small allowed values of m_S is negligible.

A particularly simple possibility for increasing the Higgs mass is to add a new quartic interaction for the Higgs doublets via the superpotential interaction $\lambda S H_1 H_2$. However, to maintain perturbative unification of the gauge couplings, a clear success of weak scale supersymmetry, the value of λ is limited. With minimal matter content this interaction provides at most an additional $\sim 10\,\text{GeV}$ to the Higgs boson mass, leading to only a small allowed region of parameter space, even including mixing amongst the Higgs bosons. In contrast, with additional matter the perturbative evolution of couplings allows a larger value of λ, increasing the Higgs boson mass by up to $\sim 20\,\text{GeV}$ compared to the theory without the singlet field. Furthermore, in this case mixing can augment the Higgs boson mass by another 2–8 GeV, considerably enlarging the allowed region of parameter space.

The extra matter implies that the gauge couplings are larger in the UV, and the top coupling smaller, compared to the minimal matter case. Providing the extra matter fills complete SU(5) multiplets, the successful unification of gauge couplings at 1 loop is

unaltered. The changes from 2 loops and threshold corrections depends on the nature of the extra matter. In the case of $5 + \bar{5}$ representations, the prediction for $\alpha_s(M_Z)$ from 2 loop running is decreased, improving the agreement with data, but this is offset by an expected increase in $\alpha_s(M_Z)$ from threshold corrections from non-degeneracies within the $5 + \bar{5}$ multiplets. The situation with $10 + \bar{10}$ representations is the opposite: 2 loop running increases the discrepancy with data, but is countered by the threshold corrections. In either case, the significance of gauge coupling unification is comparable to the standard case with minimal matter.

In this work we have considered PQ SUSY — a version of the NMSSM that incorporates the above mechanism for enlarging the Higgs mass and is fully realistic, with a minimum number of parameters. The superpotential is assumed to be exactly invariant under a Peccei-Quinn symmetry, elegantly solving the μ problem, while the soft scalar interactions include small PQ breaking interactions to give a mass to the pseudo-Goldstone boson G. The theory possesses just two parameters more than the MSSM — one is the soft mass parameter for the scalar S, m_S^2, and the other is the mass for G. A combination of vacuum stability and chargino mass limits implies a restricted range for m_S^2, so that a $10 - 20\%$ fine tuning is necessary, and a low messenger scale is preferred.

The Higgs boson system has a few characteristic properties in its spectrum and in its couplings. The spectrum contains two CP-even neutral scalars relatively close in mass, one above and one below the "naive" LEP bound of $115\,\mathrm{GeV}$ by $10 \div 20\,\mathrm{GeV}$, and with a shared coupling to the vector boson pairs, VV. Only the sum of these coupling squared is close to the squared coupling of the SM Higgs boson to VV. Related to the approximate Peccei-Quinn symmetry, the Higgs boson spectrum also contains a CP-odd light state, G, present in the main decay modes of all the CP even neutral scalars. G itself decays to $b\bar{b}$ and $\tau\bar{\tau}$, with branching ratios close to those of a light SM Higgs boson. Quite clearly, to assess the discovery potential of such a Higgs boson system at the LHC or TeVatron requires, and deserves, a detailed examination.

A small region of parameter space where $2m_\tau < m_G < 2m_b$ and G decays into $\tau^+\tau^-$ is also allowed; in this case the model predicts $BR(\Upsilon \to \gamma G)$ within reach of the existing B-factories.

The general phenomenology of the model crucially depends on the properties of the lightest neutralino, which is predicted to have a mass near $50\,\mathrm{GeV}$. Naturalness considerations suggest a low scale for supersymmetry breaking, $\sqrt{F}$, so that the gravitino is the LSP and χ_1 the next-to-LSP. In this case, all superpartner production events at colliders will yield at least two χ_1, with each decaying predominantly into a gravitino and a pseudo-Goldstone G, with a width of order $10^{-5}\,\mathrm{eV}$ for $\sqrt{F} = 100\,\mathrm{TeV}$. Given the scaling $\Gamma(\chi_1) \propto 1/F^2$, χ_1 could therefore decay inside the detector if $\sqrt{F}$ is less than of order $1000\,\mathrm{TeV}$. Pair production of χ_1 at LEP2 has a cross section of order 0.1 pb, leading in this case to events with $4\,b$ jets and missing energy. We do not know if searches by the LEP experiments would have detected this signal. If not, the generic superpartner production signal at LHC/TeVatron may include $4\,b$ jets together with the missing energy. The naturalness argument by itself is not sufficiently tight to prefer χ_1 decays inside rather than outside the detector.

Finally, the consistency of our model with the ElectroWeak Precision Tests merits further work; in particular, a non-negligible correction to the T parameter can be induced by values of λ in the region of 0.6–0.8.

It is important to note that there are alternative versions of the NMSSM with extra matter that incorporate both an enhanced Higgs boson mass, perturbative gauge coupling unification and a solution to the μ problem. The absence of mass parameters in the superpotential may be guaranteed by an R symmetry, that nevertheless allows the interaction $(\kappa/3)S^3$ as well as λSH_1H_2. In order that the S^3 interaction not substantially reduce the Higgs mass, the weak scale value of κ should be less than about 0.1. However, the form of the renormalization group equations allows κ at the unification scale to be close to unity, so this is not a powerful constraint on the theory. What is the form of the R symmetry breaking in the supersymmetry breaking scalar interactions? If A_λ is the only significant R breaking parameter, and $m_S^2 > 0$, then this theory is a perturbation of the model discussed in this paper. On the other hand there is a new minimum for $m_S^2 < 0$ that is very different from the one examined here, where the S^3 interaction prevents runaway behavior for v_s. There are also models of both the PQ and R types with large values of the symmetry breaking in the soft scalar interactions, but in these cases there are several more parameters that enter the phenomenology. Nevertheless, these models may be of interest since they may remove the need to tune m_S^2 to small values.

Note added. After completion of this work we became aware of the work of P. Schuster and N. Toro [26] where the NMSSM in the PQ and in the R-symmetric limits is analyzed with special emphasis on the fine tuning issue. We believe that the present work usefully complements ref. [26] in many different aspects.

Acknowledgments

A. 2-loop beta functions

For the convenience of the reader we give here 2-loop supersymmetric beta functions used

to produce results in section 2. The gauge coupling beta functions are:

$$\frac{dg_i}{d\log\mu} = \frac{1}{16\pi^2}b_i g_i^3 + \frac{1}{(16\pi^2)^2}g_i^3\left(\sum_{j=1}^{3} b_{ij}g_j^2 - b_{i;top}y_t^2 - b_{i;\lambda}\lambda^2\right),$$

$$b_i = \begin{bmatrix} \frac{33}{5}+n_5 \\ 1+n_5 \\ -3+n_5 \end{bmatrix}, \quad b_{i;top} = \begin{bmatrix} \frac{26}{5} \\ 6 \\ 4 \end{bmatrix}, \quad b_{i;\lambda} = \begin{bmatrix} \frac{6}{5} \\ 2 \\ 0 \end{bmatrix},$$

$$b_{ij} = \begin{bmatrix} \frac{199}{25}+\frac{7}{15}n_5 & \frac{27}{5}+\frac{9}{5}n_5 & \frac{88}{5}+\frac{32}{15}n_5 \\ \frac{9}{5}+\frac{3}{5}n & 25+7n_5 & 24 \\ \frac{11}{5}+\frac{4}{15}n_5 & 9 & 14+\frac{34}{3}n_5 \end{bmatrix}.$$

Our result for b_{ij} agrees with [25]. The dependence of b_{ij} on n_5 as given in [8] is wrong. The relevant beta functions of y_t and of the NMSSM couplings λ,κ are

$$\frac{dy_t}{d\log\mu} = \frac{y_t}{16\pi^2}\left(6y_t^2+\lambda^2-\frac{13}{15}g_1^2-3g_2^2-\frac{16}{3}g_3^2\right) - \frac{y_t}{(16\pi^2)^2}(22y_t^4+3\lambda^2y_t^2+3\lambda^4+2\kappa^2\lambda^2),$$

$$\frac{d\lambda}{d\log\mu} = \frac{\lambda}{16\pi^2}\left(4\lambda^2+3y_t^2+2\kappa^2-g_1^2-3g_2^2\right) - \frac{\lambda}{(16\pi^2)^2}(10\lambda^4+9y_t^2\lambda^2+9y_t^4+8\kappa^4+12\lambda^2\kappa^2),$$

$$\frac{d\kappa}{d\log\mu} = \frac{\kappa}{16\pi^2}(6\kappa^2+6\lambda^2) - \frac{\kappa}{(16\pi^2)^2}(24\kappa^4+24\lambda^2\kappa^2+12\lambda^4+18y_t^2\lambda^2).$$

In our analysis we have omitted the two loop contributions of gauge couplings to the running of y_t,λ,κ. This is legitimate since g_i do not approach non-perturbative values in the UV.

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